1 The diagram shows a regular octagon *ABCDEFGH*.

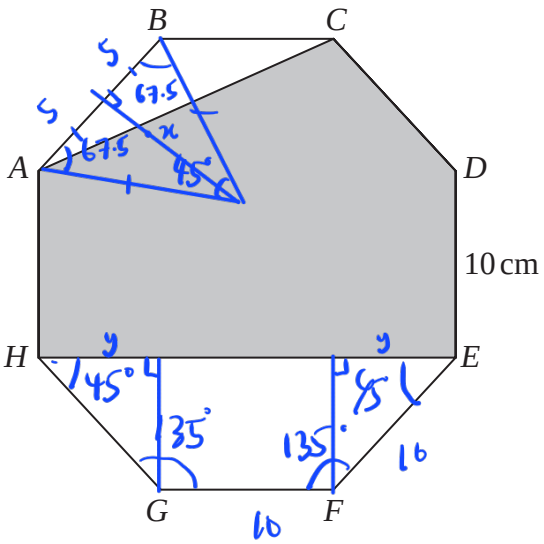


Diagram **NOT** accurately drawn

Each side of the octagon has length 10 cm.

Find the area of the shaded region *ACDEH*.
Give your answer correct to the nearest cm^2

Interior angle of octagon $= \frac{(8-2)}{8} \times 180° = 135°$ (1)

Split octagon into 8 triangles

Find x: $x = 5 \tan 67.5° = 12.07106\ldots$

Area of triangle $= \frac{1}{2} \times 10 \times 12.07106\ldots = 60.355\ldots$

Total area of octagon = 8 x 1 area of triangle

Area of octagon $= 8 \times 60.355\ldots$

$= 482.84\ldots$ (1)

Area of triangle ABC $= \frac{1}{2} \times 10 \times 10 \times \sin 135° = 25\sqrt{2} = 35.355\ldots$ (1)

Find y: $y = 10 \cos 45° = 5\sqrt{2}$

Length of HE: $2 \times 5\sqrt{2} + 10 = 10\sqrt{2} + 10$ (1)

Area of trapezium: $\frac{1}{2} \times (10\sqrt{2} + 10 + 10) \times 10 \sin 45°$

$= 120.71\ldots$ (1)

Area of shaded region = Area of octagon − area of triangle ABC − Area of trapezium

Area of shaded region = 482.84... − 35.355... − 120.71...

= 326.77...

= 327 cm² (nearest cm²) (1)

327 ..... cm²

(Total for Question 1 is 6 marks)

2 The diagram shows two circles such that the region **R**, shown shaded in the diagram, is the region common to both circles.

Area of sector:

$\frac{\theta}{360^\circ} \times \pi r^2$

Area of triangle:

$\frac{1}{2} ab \sin C$

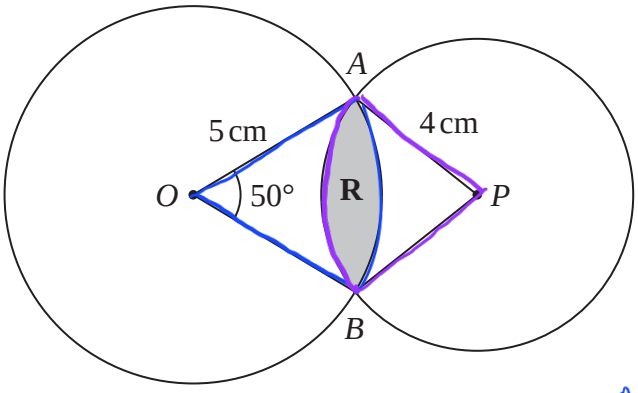
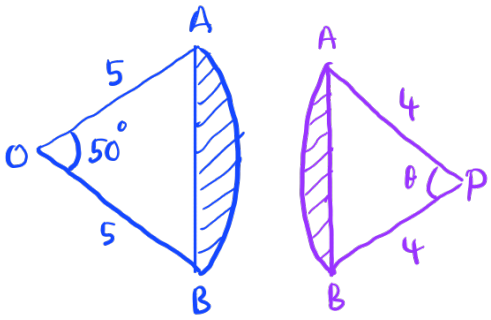


Diagram **NOT** accurately drawn

One of the circles has centre O and radius 5 cm.
The other circle has centre P and radius 4 cm.
Angle $AOB = 50°$

Calculate the area of region **R**.
Give your answer correct to 3 significant figures.



Finding length of AB (using cosine rule):

$AB^2 = 5^2 + 5^2 - 2(5)(5)\cos 50^\circ$

$AB^2 = 17.86...$

$AB = 4.226...$ (1)

Finding angle APB using known length of AB:

$4.226...^2 = 4^2 + 4^2 - 2(4)(4)\cos\theta$

$\cos\theta = \frac{4.226^2 - 4^2 - 4^2}{-2(4)(4)}$

$\cos\theta = 0.4418...$

$\theta = \cos^{-1} 0.4418...$

$= 63.78^\circ...$ (1)

Segment Area = Sector Area - Triangle Area

Large circle :

Segment Area : $\frac{50^\circ}{360^\circ} \times \pi \times 5^2 - \frac{1}{2}(5)(5)\sin 50^\circ$

$= 10.908\ldots$ (1) $- 9.576\ldots$

$= 1.332\ldots$

Small circle :

Segment Area $= \frac{63.78^\circ}{360^\circ} \times \pi \times 4^2 - \frac{1}{2}(4)(4)\sin 63.78^\circ$

$= 8.905\ldots$ (1) $- 7.1768\ldots$

$= 1.728\ldots$

Total segment area : $1.332\ldots + 1.728\ldots$ (1)

$= 3.06$ (1)

3.06 ..... cm^2

(Total for Question 2 is 6 marks)

3 The diagram shows a rectangle *ABCD* and a semicircle with diameter *AB* where *AB* = 12 cm. The point *E* lies on *DC* and also on the semicircle.

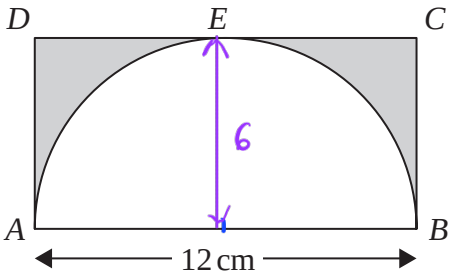


Diagram **NOT** accurately drawn

radius = 6 cm

Work out the area of the shaded region.
Give your answer correct to 3 significant figures.

Area of rectangle : $12 \times 6 = 72$ cm^2 (1)

Area of semicircle : $\frac{1}{2} \times \pi \times 6^2 = 56.54$ cm^2

Area of shaded region = Area of rectangle − Area of semicircle

: 72 cm^2 − 56.54 cm^2 (1)

: 15.5 cm^2 (1)

15.5 ..... cm^2

(Total for Question 3 is 3 marks)

4 *A*, *B* and *C* are points on a circle with centre *O*.

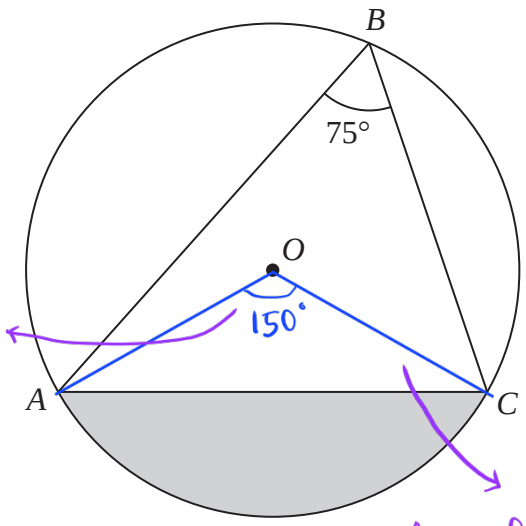


Diagram **NOT** accurately drawn

Angle at Centre is twice the angle at circumference

$75 \times 2 = 150°$ (1)

Area of triangle $= \frac{1}{2}ab\sin C$

Angle $ABC = 75°$

The area of the shaded segment is $200\,cm^2$

Calculate the radius of the circle.
Give your answer correct to 3 significant figures.

(1) $\frac{150°}{360°} \times \pi r^2 = \frac{1}{2} r^2 \sin 150° + 200$ (1)

$\frac{5\pi r^2}{12} = \frac{1}{2} r^2 \left(\frac{1}{2}\right) + 200$

$= \frac{1}{4} r^2 + 200$

$\frac{5\pi}{12} r^2 - \frac{1}{4} r^2 = 200$ (1)

$(1.0589\ldots) r^2 = 200$

$r^2 = \frac{200}{1.0589\ldots}$

$= 188.85\ldots$

$r = \sqrt{188.85\ldots}$

$r = 13.7$ (3sf)

13.7 (1)

..... cm

(Total for Question 4 is 5 marks)

5 *A*, *B* and *C* are points on a circle with centre *O*.

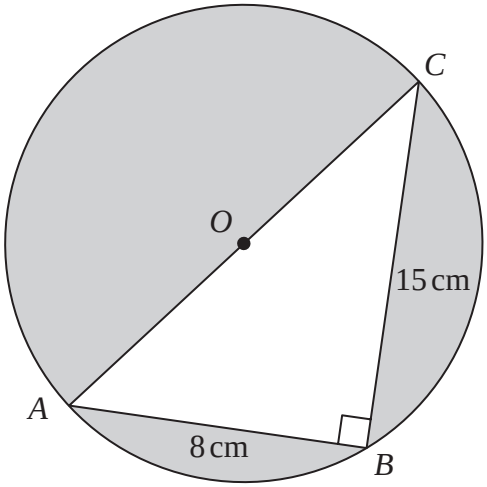


Diagram **NOT** accurately drawn

AOC is a diameter of the circle.

AB = 8 cm *BC* = 15 cm

Angle *ABC* = 90°

Work out the total area of the regions shown shaded in the diagram.
Give your answer correct to 3 significant figures.

Area of triangle $= \frac{1}{2}ab\sin C$

Area of triangle $= \frac{1}{2} \times 8 \times 15 \times \sin 90°$

$= 60$

$AC = \sqrt{8^2 + 15^2}$ (1)

$= 17$ (1)

radius of circle $= 17 \div 2 = 8.5$ cm

Area of circle $= \pi r^2$

$= \pi (8.5)^2$

$= 226.98$ (1)

Area of shaded region $= 226.98 - 60$ (1)

$= 166.98$

$= 167$ (3sf) (1)

167 ..... cm^2

(Total for Question 5 is 5 marks)

6

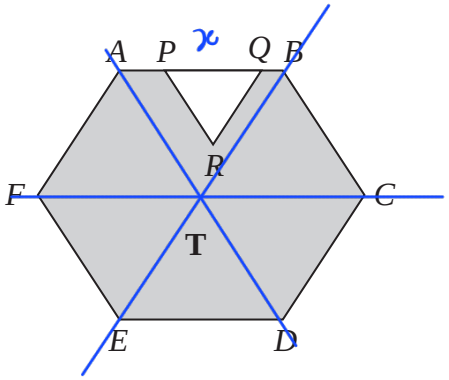


Diagram **NOT** accurately drawn

The diagram shows a shaded region **T** formed by removing an equilateral triangle *PQR* from a regular hexagon *ABCDEF*.

The points *P* and *Q* lie on *AB* such that $AB = 1.5 \times PQ$

Given that the area of region **T** is $72\sqrt{3}$ cm^2

work out the length of *PQ*.

$AB = x$

Area of one triangle in hexagon $= \frac{1}{2} ab \sin C$

$= \frac{1}{2} x^2 \sin 60°$

$= \frac{\sqrt{3}}{4} x^2$ (1)

Area of hexagon $= 6 \times \frac{\sqrt{3} x^2}{4}$

$= \frac{3\sqrt{3}}{2} x^2$ (1)

Area of PQR $= \frac{1}{2} ab \sin C$

$= \frac{1}{2} \left(\frac{2}{3}x\right)^2 \sin 60°$

$= \frac{\sqrt{3}}{9} x^2$

Area of shaded region $= \left(\frac{3\sqrt{3}}{2} - \frac{\sqrt{3}}{9}\right) x^2$

$72\sqrt{3} = \frac{25\sqrt{3}}{18} x^2$ (1)

$x^2 = \frac{18 \times 72\sqrt{3}}{25\sqrt{3}}$

$= \frac{1296}{25}$

$x = \sqrt{\frac{1296}{25}}$

$x = \frac{36}{5}$

$PQ = \frac{2}{3} AB$

$= \frac{2}{3} \times \frac{36}{5}$

$= \frac{24}{5}$

$= 4.8$ (1)

4.8 ..... cm

(Total for Question 6 is 4 marks)

7 The diagram shows four identical circles drawn inside a square.

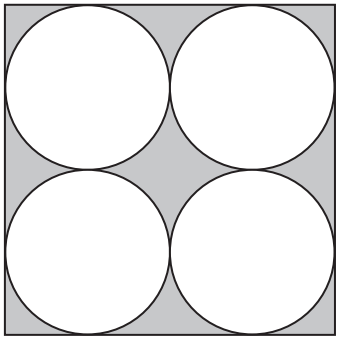


Diagram **NOT** accurately drawn

Each circle touches two other circles and two sides of the square.

The region inside the square that is outside the circles, shown shaded in the diagram, has a total area of 40 cm^2

Work out the perimeter of the square.
Give your answer correct to 3 significant figures.

Area of square $= 4r \times 4r = 16r^2$ (1)

Area of 4 circles $= 4 \times \pi \times r^2 = 4\pi r^2$

$16r^2 - 4\pi r^2 = 40$

$r^2 = \dfrac{40}{16 - 4\pi}$

$r = \sqrt{\dfrac{40}{16 - 4\pi}} = 3.413\ldots$ (1)

Perimeter $= 16 \times 3.413\ldots$ (1)

$= 54.6$ (1)

54.6 ..... cm

(Total for Question 7 is 4 marks)

8 Jonty has a storage container in the shape of a cuboid, as shown in the diagram.

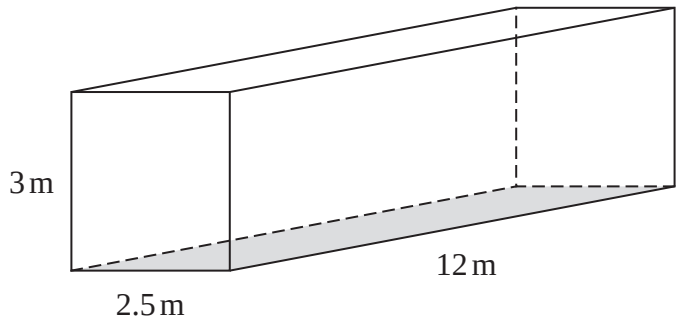


Diagram **NOT** accurately drawn

Jonty is going to paint the outside of his storage container, apart from the base which is shown shaded in the diagram.
He needs enough paint to cover the four sides and the top.

Each tin of paint covers an area of 15 m^2

The cost of each tin of paint recently increased by 10%
After the increase, the cost of each tin of paint is £26.95

Jonty says

"**Before** the increase, I could have bought enough paint for less than £200"

Show that Jonty is correct.
Show your working clearly.

Area : $3 \times 2.5 = 7.5$ (1)

$12 \times 3 = 36$

$12 \times 2.5 = 30$

Total area : $(2 \times 7.5) + (2 \times 36) + 30$

$= 15 + 72 + 30$ (1)

$= 117$

Tin of paint needed = $\frac{117}{15} = 7.8$ (1)

$\approx$ 8 tins are needed

100% + 10% = 110% which is £26.95 (1)

Price at 100% : $x = \frac{26.95}{110} \times 100$

(1)

$= 24.5$

24.5×8 tins $= 196$

Yes. Jonty is correct. (1)

(Total for Question 8 is 6 marks)

9 *A*, *B* and *C* are points on a circle, centre *O*

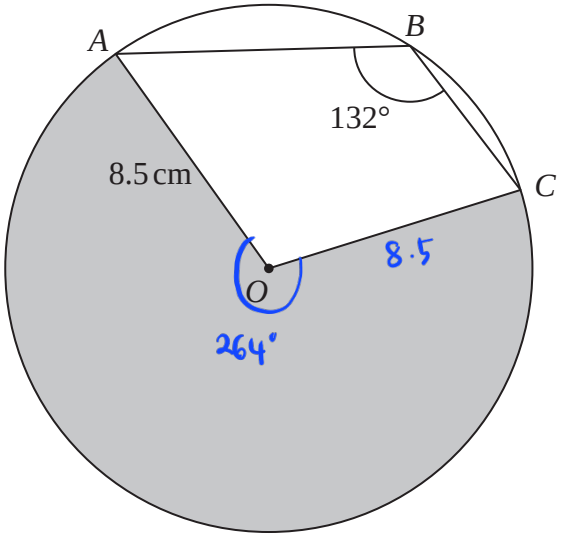


Diagram **NOT** accurately drawn

The radius of the circle is 8.5 cm
Angle *ABC* = 132°

Work out the perimeter of the shaded sector *AOC*
Give your answer correct to 3 significant figures.

$\angle AOC = 132^\circ \times 2 = 264^\circ$ (1)

$\frac{264}{360} \times 2 \times \pi \times 8.5 = 39.1\ldots$ (1)

Perimeter : $39.1\ldots + 8.5 + 8.5$

$= 56.2$ (1)

56.2 ..... cm

(Total for Question 9 is 3 marks)

10 The diagram shows the cross section of a circular water pipe.

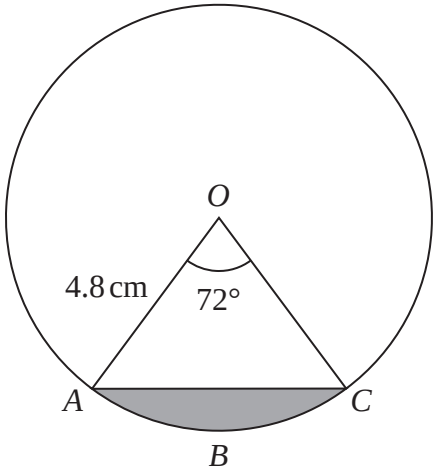


Diagram **NOT** accurately drawn

OABC is a sector of the circle, centre *O*

The shaded region in the diagram represents the water flowing in the pipe.

The water flows at 14 cm/s in the pipe.

Work out the volume of water that has flowed through the pipe in 3 minutes.
Give your answer in cm^3 correct to 3 significant figures.

Area of sector : $\pi \times 4.8^2 \times \frac{72}{360} = 14.476\ldots$ (1)

Area of triangle : $\frac{1}{2} \times 4.8^2 \times \sin 72 = 10.956\ldots$ (1)

Area of shaded : $14.476\ldots - 10.956\ldots$ (1)

$= 3.520\ldots$

Volume : $3.520\ldots \times 14\ cm/s \times (3 \times 60)s$ (1)

$= 3.520\ldots \times 2520$

$= 8870$ (1)

8870 ..... cm^3

(Total for Question 10 is 5 marks)

11 The diagram shows two circles with centre O and a regular pentagon $ABCDE$

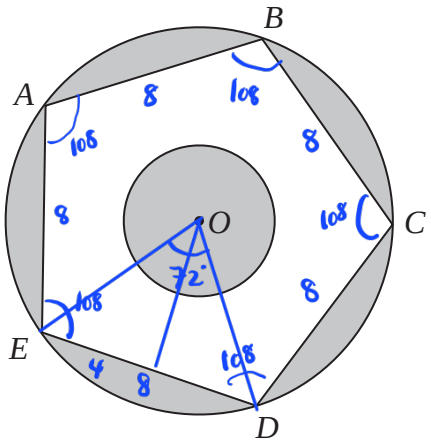


Diagram **NOT** accurately drawn

A, B, C, D and E are points on the larger circle.
The pentagon has sides of length 8 cm.

The diagram is shaded such that

shaded area = unshaded area

Work out the radius of the smaller circle.
Give your answer correct to 3 significant figures.

pentagon angle $= \frac{180 \times 3}{5} = 108°$

angle EOD $= 180 - 54 - 54$
$= 72°$

height of triangle; $\tan 54 = \frac{\text{height}}{4}$
$= 4 \tan 54 = 5.505\ldots$ (1)

$\frac{\text{length OE}}{\sin 54°} = \frac{8}{\sin 72}$

$OE = \frac{8 \sin 54}{\sin 72} = 6.805\ldots$ = radius of large circle

Area of whole diagram $= \pi \times 6.805^2 = 145.489\ldots$ (1)

Area of pentagon $= 5 \times \frac{1}{2} \times 8 \times 5.505\ldots = 110.11$ (1)

Shaded area = unshaded area

$145.489 - 110.11 + \pi r^2 = 110.11 - \pi r^2$ (1)

$2\pi r^2 = 74.731\ldots$ (1)

$r^2 = 11.89\ldots$

$r = 3.45$ (3 s.f.) (1)

3.45 ..... cm

(Total for Question 11 is 6 marks)